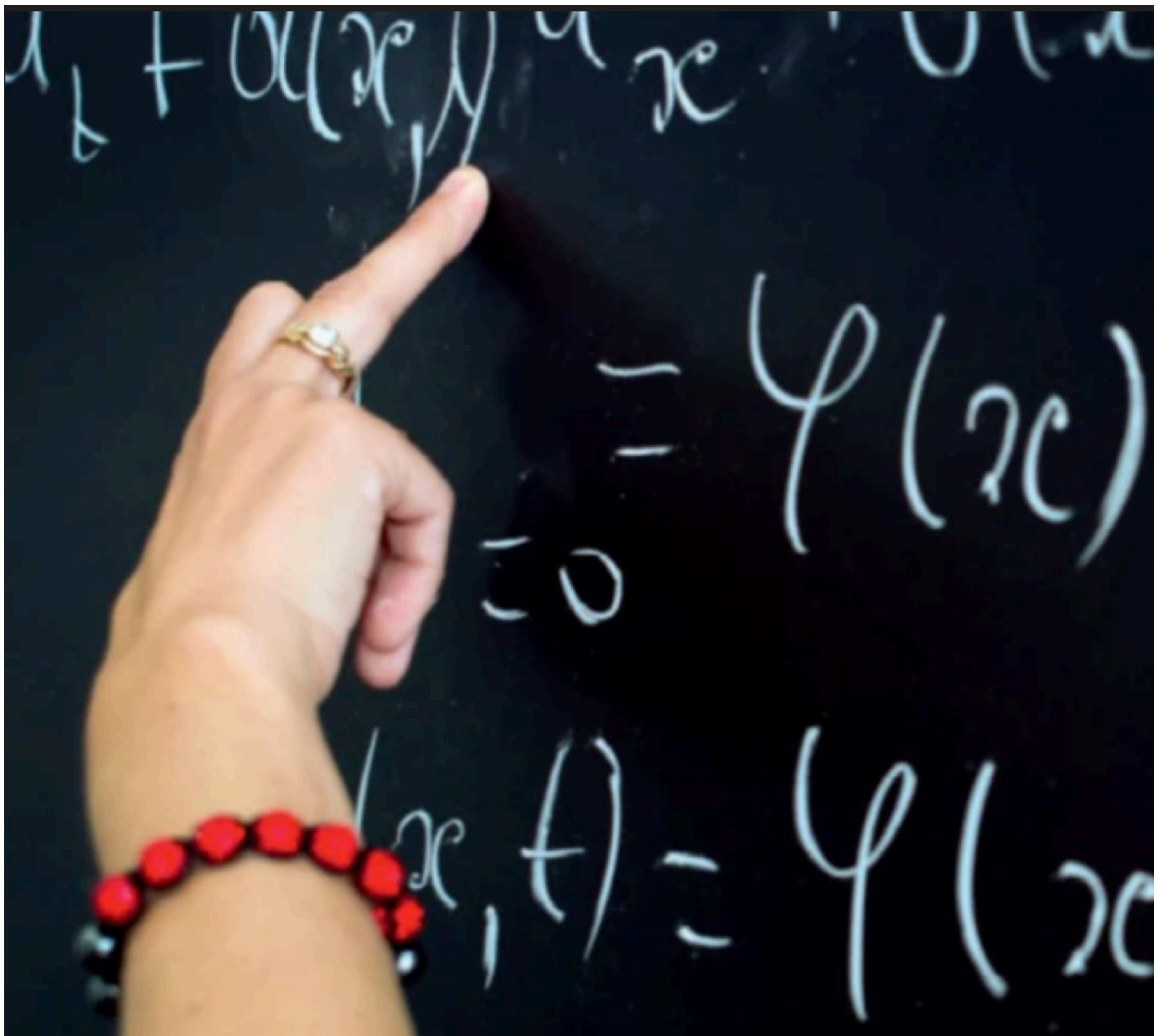
The être of Michèle Vergne



Arun Ram
University of Melbourne

29 August 2019
Women in Maths Day
University of Melbourne

Michèle Vergne

Michèle Vergne

all of mathematics

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all of mathematics

The Universe

So, … I began to prepare a 15min talk about

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How can I give my audience a clear and serious feel for the depth and content of the Universe, … in 15 min?

Step 1: **Google Michèle Vergne**

Step 2: **The Wikipedia page**

Step 3: **Michèle's home page**

Step 4: **Rédaction des cours**

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Focus the talk… focus the talk…. focus the talk ….

The “locals”

Step A: **Google ‘Marcy Kashiwara Vergne’**

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The Kashiwara-Vergne conjecture

The “locals”

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YES! the focus is …

The Kashiwara-Vergne conjecture

C. Torossian's Séminaire Bourbaki article **on**

The Kashiwara-Vergne conjecture

C. Torossian's Séminaire Bourbaki article

Walter Rudin: Real and Complex Analysis

"This is the most important function in mathematics."

$$e^x = 1 + x + \frac{1}{2}x^2 + \frac{1}{6}x^3 + \frac{1}{24}x^4 + \frac{1}{120}x^5 + \frac{1}{720}x^6 + \cdots$$

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If $xy = yx$ **then**

$$e^x e^y = e^{x+y}\,.$$

$$
\begin{aligned}
e^{x+y} &= 1 + (x+y) + \frac{1}{2}(x+y)^2 + \frac{1}{6}(x+y)^3 + \cdots \\
&=
\end{aligned}
$$

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$$
\begin{array}{ccccccc}
= & & & 1 & & & \\
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$$= \quad e^x + e^x y + e^x \frac{1}{2} y^2 + e^x \frac{1}{6} y^3 + \cdots$$

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\end{array}$$

$$= \quad e^x + e^x y + e^x \frac{1}{2} y^2 + e^x \frac{1}{6} y^3 + \cdots \quad = e^x e^y .$$

The basis of Lie theory, mathematical physics, probability, and a few other subjects is

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when x is a matrix.

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RATHER

When x and y are matrices

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RATHER

$$e^x e^y = e^{\left(\begin{array}{c} x + y + \frac{1}{2}[x, y] + \frac{1}{12}[x, [x, y]] + \frac{1}{12}[y, [y, x]] \\ + \frac{1}{48}[y, [x, [y, x]]] + \frac{1}{48}[y, [x, [y, x]]] + \cdots \end{array} \right)}$$

where $[a, b] = ab - ba\,.$

C. Torossian's Séminaire Bourbaki article

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In shorter form,

$$e^x e^y = e^{x+y+Z(x,y)}$$

The Kashiwara-Vergne conjecture says to write

$$Z(x,y) = (1-e^{ad_x})F(x,y) + (1-e^{ad_y})G(x,y)$$

and they tell you what they expect *F(x,y)* and *G(x,y)* to be.

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The trace condition is an important part of the "understanding" and "control" of the formula:

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DISTRACTION from talk preparation

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so, for example, $\zeta(2) = \sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$.

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where,
in ardent anticipation
inviting us to what is to come,
Michèle has polished the mirrors,
chandeliers and gilded banisters
to an exciting and expectant shine.

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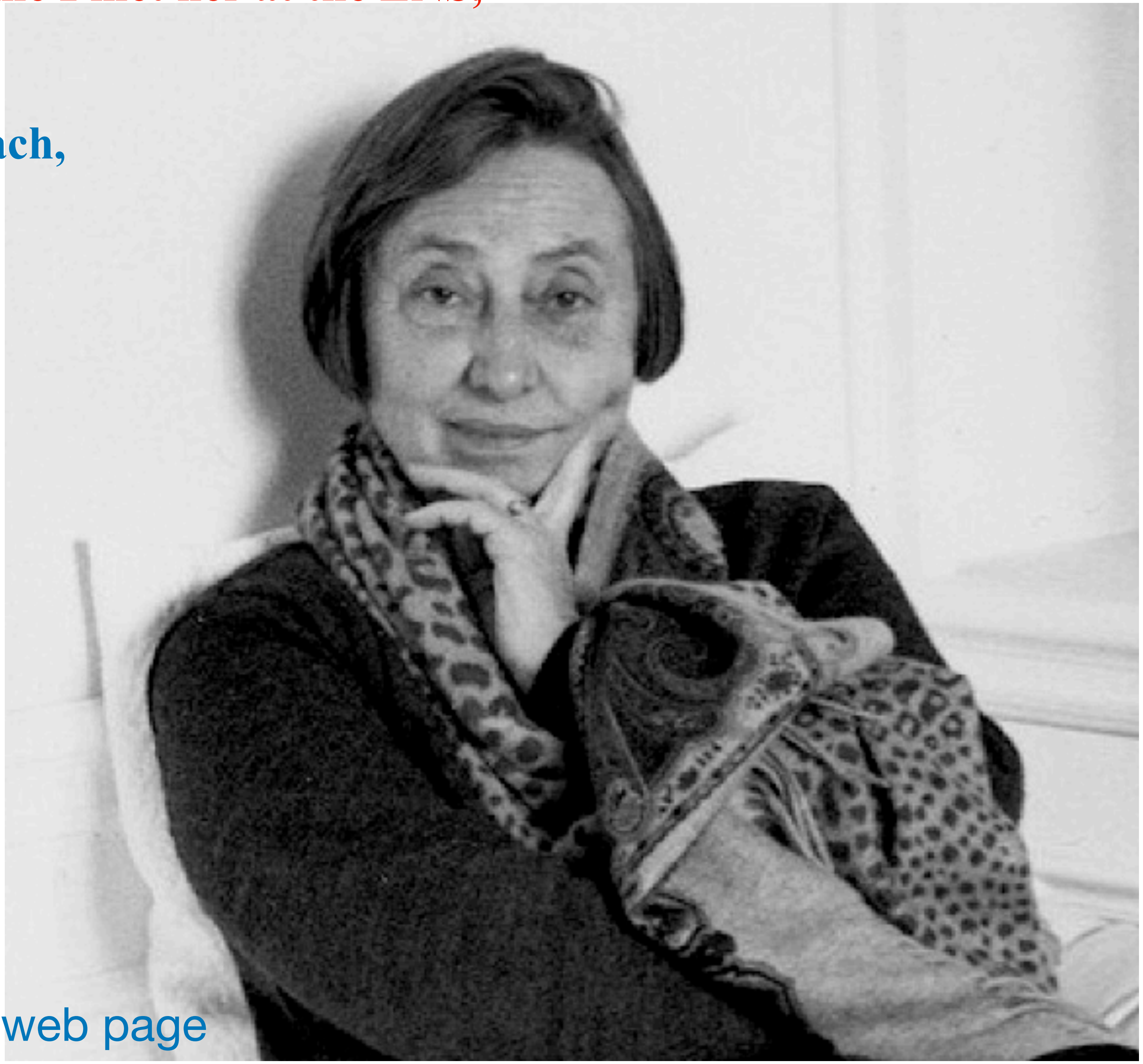

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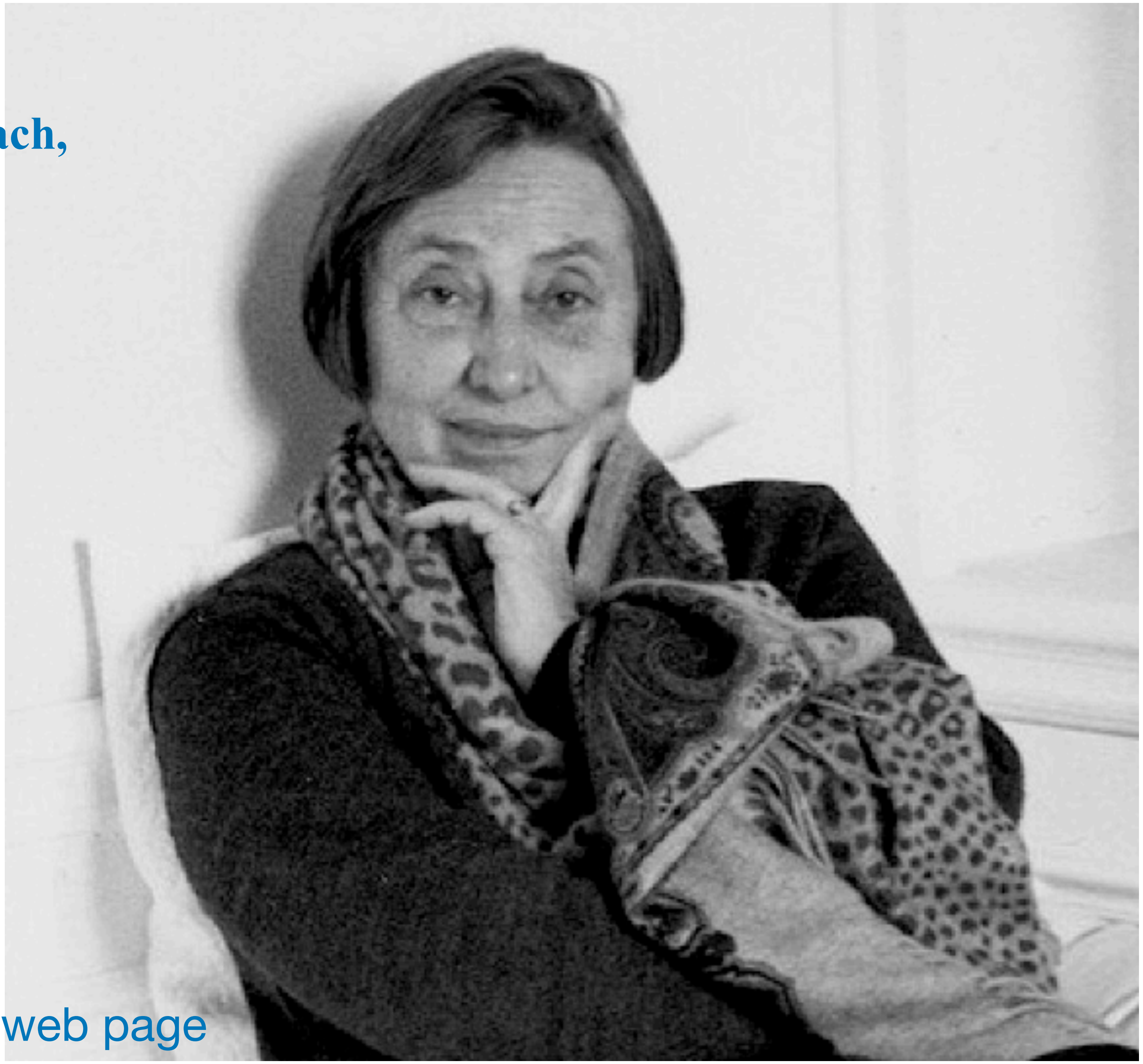

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Rudin says e^x is the most important function in mathematics

Michèle Vergne taught us what $e^x e^y$ is.

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I did not practice the end of this talk because each time I tried I found it difficult to retain composure. These are the words of Michèle remembering Malliavin from the Notices of the AMS. For me, the feeling when I read this the same as when I read her mathematics: her être is so consuming that it is a covering space of my own existence and its envelopment in hers. Please allow me to try, so that I may attempt to share this special experience with you.

Malliavin and I
by
Michèle Vergne

Notices of the American Mathematical Society, April 2011

Pisa, June 2010: I am in Pisa, I learn via email about the death of Paul Malliavin. If anyone seemed to me to be immortal among immortals, it was he. I think of him introducing me to the Académie and guiding me on a visit to the Institut Library and the Bibliothèque Mazarine. We could see the Seine through the windows. He told me about Cardinal Richelieu, to whom I think he attributed the sentence: "I shall regret the beauty of this place when in the other world".

I.H.P., May 1968: I am twenty-five-years old, long hair, in blue jeans. My heroine is Louise Michel. I can well see myself sent to hard labor for my ideas.

Slogans, demonstrations: the system must be changed. The dusty “Institut Henri Poincaré” must be destroyed. General meetings, declarations. When Malliavin intervenes with his soft voice, I shout: “Malliavin is a bourgeois, Malliavin is a fascist”. If the red guard were gathering their battalions, I would be with them and would send Malliavin out to serve the people. Malliavin continues to smile serenely. Ever since then, when I meet him on the streets of Paris, he tips his hat and addresses me with a ceremonious: “Chère Madame”.

At the Theater, January 2009: Malliavin is presenting a candidate for membership in the Académie des Sciences. It won't be an easy win. I sit beside him at the green, oval desk. He draws a few crumpled sheets of paper out of his pocket. "Madame, I would like to have your opinion of my speech," and, in a low voice, he starts to whisper: "Already a hundred years ago, Elie Cartan…." Then comes his turn to speak, and in a loud voice, he declaims: "Already a hundred years ago…." Bravo, clap, clap, and his candidate is elected.

The Poisson Summation Formula: Malliavin and I are ecstatic about the Poisson summation formula. No doubt, he knows all its finest and deepest aspects. I don't, but I nonetheless think it is the most beautiful formula in mathematics.

Two Things That Malliavin Loves, Mathematics and Influencing People's Destiny: These are not unrelated. Speaking about a colleague, he often lauds the beauty of his work: "Demailly's annulation theorem is extraordinary; Madame, consider that it does not require pointwise estimates, but only in the mean. . . ." Villani's work overjoys him.

A Reception at the Malliavins' Home: I am invited to a reception at the Malliavins' home. I go with my daughter Marianne. She was eight years old at the time. We enter a paved courtyard, we go up some stairs, we ring a bell, and we enter an apartment, immersed in semi-darkness.

Overflowing bookshelves cover the walls; the seats are antique, I fear that the furniture would disintegrate into dust were the curtains to be drawn.

Malliavin talks to my daughter, he finds an old, illustrated edition of *The Children of Captain Grant* for her. She sits on a window sill and reads passionately, while the other guests, mainly mathematicians from all over the world, tell each other about their lives. Marie-Paule becomes nervous: the petits fours must be eaten, the Berthillon sorbets must be tasted. . . .

Temptations: Malliavin phones me, he wants to propose me as a candidate for the Académie. I object: “My father and mother are dead, it is too late to please them, it would give me no personal pleasure.” Malliavin responds: “Madame, we are not Académicians for our pleasure, but to serve our country”. He then invites me to come to the Académie and leads me to the *salle des séances*. In the dim light, the white marble heads observe the scene. That night, I have a dream: there is a lit niche, and inside the niche, a bottle of whiskey sitting on a pedestal (I had just seen again *Rio Bravo*). I realize that, more than anything else, I wish to have a draught of whiskey and that I would also like to enter the Académie. I phone Malliavin: “Yes, I agree to be nominated.” Anyway, I am totally incapable of saying *no* to Malliavin.

No: I am elected. My sister Gilberte does not survive for my election, and my sister Martine is about to die. Now other plans are afoot behind the scene: a representative of France for the executive committee of the International Mathematical Union is needed. Malliavin and Jacques-Louis Lions contact me, they have decided it should be me. Malliavin calls me daily. “No, I do not want to, I cannot.” After each phone call, a wave of anxiety suffocates me. I feel like a fraud.

Honors: I become accustomed to taking pleasure in honors. Today is the *séance solennelle* (solemn convocation) at the Académie of Sciences. Going down the stairs between the raised sabers of the republican guards seems natural.

Malliavin is wearing his ceremonial outfit. Befittingly, the Académie's paleontologist has a sword with a dinosaur pommel. Malliavin is happy to be among his peers. He knows them all. He pushes a chair forward for Pierre Lelong, who is nearly ninety. He listens politely to Denisse, he teases Dercourt, he says a kind word to me. He jokes: "Here, we all love researchers studying longevity and who search for a happiness pill to give the elderly"; and, all the while, he is predicting the election of Beaulieu [who specializes in geriatrics] as the new president of the Académie des Sciences.

Maneuvers: Malliavin has a plan for *X*: he sends stacks of mail, phones, counts his cards. He scrutinizes the weaknesses of the opposition's plans for *Y*. If the maneuver fails, it's a triumph for *Z*, and *Y* is chosen. Malliavin gives me sibylline advice. I interpret it as follows: as soon as someone is chosen for our section, we should all forget whatever bad things we once thought about him.

Should we do likewise with the dead? It might be wise, since we will have to spend eternity by their sides.

Haar Measures and Malliavin Measures: Is there some sort of Haar measure for loop groups? Which are the “natural” groups that admit unitary representations? Locally compact groups, thanks to Haar measure, but the unitary group also has a unitary representation. We may also construct ergodic measures for some natural groups, such as the infinite permutation group. These are questions that interest Paul Malliavin and Marie-Paule.

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In the same way, for me the world without Malliavin is not quite the same. I miss him.